Convergence to second order stationary points in nonlinear programming

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Let us be given the nonlinear programming problem $P : \{\min f(x) : g(x) \leq 0\}$, where $f : \Re^n \rightarrow \Re$ and $g : \Re^n \rightarrow \Re^m$ are thrice continuously differentiable. A *second order stationary point* for problem P is a point that, toghether with a suitable multiplier $\lambda \in \Re^m$, satisfies not only the KKT *first order* necessary optimality conditions: $\nabla f(x) + \nabla g(x)\lambda = 0,\ g(x) \leq 0,\ \lambda \geq 0,\ \lambda' g(x) = 0$, but also the *second order* necessary optimality conditions:

$$\nabla g_i(x)' z = 0,\ \forall i : g_i(x) = 0 \ \Rightarrow\ z'\Big(\nabla^2 f(x) + \sum_{i=1}^{m} \lambda_i \nabla^2 g_i(x)\Big) z \geq 0.$$

We are interested in an algorithm for solving P that guarantees covergence to second order stationary points. This property makes it much more likely that the limit points of the sequence generated by the algorithm are local minimizers.

In unconstrained optimization, this kind of convergence has been obtained by using directions of negative curvature for the objective function in line search algorithms. However for the consrained problem P, very few algorithms have been described with the required property.

After a short review of existing results, we define a new algorithm that generates a sequence whose limit points are second order stationary points for P.

The algorithm is defined in the primal-dual space of the original variables x and of the multipliers λ, and is based on two main tools: a locally convergent truncated Newton-type iteration, and a globalization strategy using a continuously differentiable merit function.

More in particular, the local iteration produces two directions. A first order direction, which ensures local superlinear convergence toward a KKT pair, and a second order direction which is used to enforce convergence toward second order stationary points of the overall algorithm. These two directions are employed to compute the new primal-dual iterate by means of a linesearch procedure based on an exact augmented Lagrangian function. In this way we get a twofold result. On the one hand, we can assess the goodness of the search directions produced by the local algorithm and measure their progress toward a KKT pair; on the other hand, by exploiting the non-convexity of the merit function, iterates can escape from first-order stationary points and converge to second order ones.