Generating all vertices in implicitly defined polytopes

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The classical vertex enumeration problem is a representation conversion problem for convex polytopes, that is, to transform a halfspace representation (H-representation) of a convex polytope to its vertex representation (V-representation). In this talk, we present recent algorithmic questions for the vertex enumeration problem for implicitly defined polytopes and polyhedra where neither H- nor V-representation is available.

These include (1) the computation of the vertices of the Minkowski addition of several V-polytopes, (2) the enumeration of all Gróbner bases of a polynomial ideal and (3) the computation of the convex hull of several convex polytopes, all in general dimensional Euclidean space. We show that the reverse search scheme can be applied to these problems and lead to practical algorithms that are highly parallelizable. Recent implementations of these algorithms are also reported to indicate the power and the limits of these "super heavy" computations that require to solve a large number of small scale LPs.